

## Chapter 3 : Understanding Quadrilaterals

### Question 1

A quadrilateral has three acute angles, each measures  $80^\circ$ . What is the measure of the fourth angle?

#### Solution:

Sum of the four angles of a quadrilateral =  $360^\circ$

$$80^\circ + 80^\circ + 80^\circ + 4\text{th angle} = 360^\circ$$

$$\therefore 4\text{th angle} = 360^\circ - (80^\circ + 80^\circ + 80^\circ) = 360^\circ - 240^\circ = 120^\circ$$

### Question 2

In a quadrilateral ABCD, the measure of the three angles A, B and C of the quadrilateral are  $110^\circ$ ,  $70^\circ$  and  $80^\circ$  respectively. Find the measure of the third angle.

#### Solution:

The measure of A =  $110^\circ$

The measure of B =  $70^\circ$

The measure of C =  $80^\circ$

The sum of the four angles of the quadrilateral ABCD =  $\angle A + \angle B + \angle C$

$$+\angle D=360^\circ.$$

$$\angle A + \angle B + \angle C = 110^\circ + 70^\circ + 80^\circ = 260^\circ$$

$$\angle A + \angle B + \angle C + \angle D = 360^\circ$$

$$\angle D = 360^\circ - (\angle A + \angle B + \angle C)$$

$$= 360^\circ - 260^\circ$$

$$= 100^\circ$$

### Question 3

In a quadrilateral ABCD,  $\angle D$  is equal to  $150^\circ$  and  $\angle A = \angle B = \angle C$ . Find  $\angle A$ ,  $\angle B$  and  $\angle C$ .

#### Solution:

Measure of  $\angle D = 150^\circ$

Let  $\angle A = \angle B = \angle C = x^\circ$

Sum of the angles of the quadrilateral is  $360^\circ$ .

$$\Rightarrow x^\circ + x^\circ + x^\circ + 150^\circ = 360^\circ$$

$$\Rightarrow 3x^\circ + 150^\circ = 360^\circ$$

$$\Rightarrow 3x^\circ = 360^\circ - 150^\circ = 210^\circ$$

$$\therefore x = \frac{210^\circ}{3} = 70^\circ$$

$$\therefore \angle A = 70^\circ, \angle B = 70^\circ \text{ and } \angle C = 70^\circ.$$

#### **Question 4**

**The angles of a quadrilateral are in the ratio 1:2:3:4. What are the measures of the four angles?**

#### **Solution:**

Given the ratio of the angles of a quadrilateral = 1:2:3:4

Therefore, let the angles of the quadrilateral be  $x$ ,  $2x$ ,  $3x$  and  $4x$ .

The sum of the angles of a quadrilateral is  $360^\circ$ .

$$\Rightarrow x+2x+3x+4x = 360^\circ$$

$$\Rightarrow 10x = 360^\circ$$

$$\Rightarrow x = 36^\circ$$

$$\Rightarrow 2x = 2 \times 36^\circ = 72^\circ$$

$$\Rightarrow 3x = 3 \times 36^\circ = 108^\circ$$

$$\Rightarrow 4x = 4 \times 36^\circ = 144^\circ$$

$\therefore$  The measures of the four angles are  $36^\circ$ ,  $72^\circ$ ,  $108^\circ$  and  $144^\circ$ .

### Question 5

The In a quadrilateral

- (i) which of them have their diagonals bisecting each other?
- (ii) which of them have their diagonals perpendicular to each other?
- (iii) which of them have equal diagonals ?

#### Solution:

Diagonals bisect each other in

- a) parallelogram
- b) rhombus
- c) rectangle
- d) Square
- e) Kite

(ii) Diagonals are perpendicular in

- a) rhombus
- b) Square
- c) Kite

(ii) Diagonals are perpendicular in

- a) rhombus
- b) Square
- c) Kite

(iii) Diagonals are equal to each other in

- a) rectangle.
- b) square

### Question 6

Adjacent sides of a rectangle are in the ratio 5: 12, if the perimeter of the rectangle is 34cm, find the length of the diagonal.

#### Solution:

Given the adjacent sides of a rectangle are in the ratio 5:12.

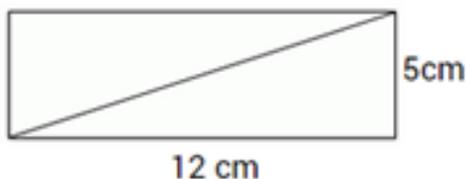
Therefore let the sides be  $5x$  and  $12x$ .

$$\text{Then } 5x + 12x + 5x + 12x = 34$$

$$34x = 34$$

$$x = 1 \text{ cm}$$

Hence the sides are 12cm and 5cm.



The length of the diagonal =  $\sqrt{(5^2 + 12^2)}$  (In a right angled triangle applying Pythagoras theorem)

$$\begin{aligned} &= \sqrt{(25 + 144)} \\ &= \sqrt{169} = 13\text{cm.} \end{aligned}$$

Therefore the length of the diagonal is 13cm.

### Question 7

The opposite angles of a parallelogram are  $(3x + 5)^\circ$  and  $(61 - x)^\circ$ . Find the measure of four angles.

**Solution:**

$(3x + 5) = (61 - x)$  (Opposite angles of a parallelogram are equal)

$$3x + x = 61 - 5$$

$$4x = 56^\circ$$

$$x = \frac{56^\circ}{4}$$

$$x = 14^\circ$$

$$3x + 5 = 3(14) + 5 = 42 + 5 = 47^\circ$$

$$61 - x = 61 - 14 = 47^\circ$$

Angle adjacent to one of the above angle =  $180^\circ - 47^\circ$

$= 133^\circ$  (Sum of adjacent angles in a parallelogram is  $180^\circ$ )

Fourth angle =  $133^\circ$  (Opposite angles of a parallelogram are equal)

Therefore the four angles in a parallelogram are  $47^\circ$ ,  $133^\circ$ ,  $47^\circ$  and  $133^\circ$

### Question 8

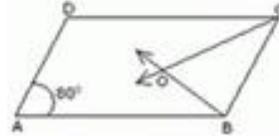
ABCD is a ||gm with  $\angle A = 80^\circ$ . The internal bisectors of  $\angle B$  and  $\angle C$  meet at O. Find the measure of the three angles of  $\Delta BCO$ .

#### Solution:

$\angle C = \angle A$  (Opposite angles of a ||gm are equal)

$\angle C = 80^\circ$  (Given  $\angle C = 80^\circ$ )

$\angle OCB = \frac{1}{2}\angle C = \frac{1}{2} \times 80 = 40^\circ$



$\angle B = 180^\circ - \angle A$  (Sum of interior angles on the same side of the transversal is  $180^\circ$ )

$$= 180^\circ - 80^\circ$$

$$= 100^\circ$$

$\angle CBO = \frac{1}{2}\angle B = \frac{1}{2} \times 100^\circ = 50^\circ$   $\angle BOC = 180^\circ - (\angle OBC + \angle CBO)$  (Angle sum of a  $\Delta$ )

$$= 180^\circ - (40^\circ + 50^\circ)$$

$$= 180^\circ - 90^\circ$$

$$= 90^\circ$$

$\therefore$  The Three angles of the triangle BCO namely  $\angle OCB$ ,  $\angle CBO$ ,  $\angle BOC$  are  $40^\circ$ ,  $50^\circ$  and  $90^\circ$  respectively.

### Question 9

Find the measure of all four angles of a parallelogram whose consecutive angles are in the ratio 1 : 3.

**Solution:**

Given consecutive angles of a parallelogram are in the ratio 1:3

Therefore, the two consecutive angles be  $x$  and  $3x$ .

$x + 3x = 180^\circ$  (sum of the interior angles on the same side of the transversal is  $180^\circ$ )

$$4x = 180^\circ$$

$$x = 45^\circ$$

Therefore the two consecutive angles are  $45^\circ$  and  $3(45^\circ) = 135^\circ$ .

Since the opposite angles of a parallelogram are equal. The measures of all four angles of a parallelogram are  $45^\circ, 45^\circ, 135^\circ$  and  $135^\circ$ .

**Question 10**

A diagonal and a side of a rhombus are of equal length. Find the measure of the angles of the rhombus.

**Solution:**

Let ABCD be the rhombus.

$AB = BC = DC = DA$  (sides of a rhombus are equal)

But  $AB = BD$  (Given)

$\therefore AB = BC = CD = DA = BD$

Since in  $\Delta ABD$  all the sides are equal.  $\Delta ABD$  is an equilateral  $\Delta$ .

Similarly  $\Delta BCD$  is also an equilateral.

$$\angle A = \angle ABD = \angle ADB = \angle DBC = \angle C = \angle CDB = 60^\circ$$

$$\therefore \angle B = \angle ABD + \angle DDC = 60^\circ + 60^\circ = 120^\circ$$

$$\text{and } \angle D = \angle ADB + \angle CDB = 60^\circ + 60^\circ = 120^\circ$$

$\therefore$  The angles of the rhombus are  $60^\circ, 120^\circ, 60^\circ$  and  $120^\circ$ .

